International Journal of Thermophysics, Vol. 24, No. 2, March 2003 (© 2003)

Note

Overshooting Phenomenon in the Hyperbolic Microscopic Heat Conduction Model

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Received May 13, 2002

The overshooting phenomenon under the effect of the microscopic hyperbolic heat conduction model is investigated. A map tracing the region within which the overshooting phenomenon occurs is presented. The two most important parameters which control the overshooting phenomenon are found to be the first and second time-derivatives of the temperature at $t=0$. However, in order for the overshooting to appear, a higher initial value of the second time-derivative of the temperature change is required than the initial value of the first time-derivative of the temperature. Overshooting is more likely to appear in the parabolic, rather than in the hyperbolic, microscopic heat conduction model.

KEY WORDS: hyperbolic microscopic model; microscopic heat conduction; overshooting; two-step hyperbolic model; two-step heat conduction model.

1. INTRODUCTION

Temperature overshooting is concerned with the excess temperature established in a conducting medium when two thermal wavefronts meet. The overshooting phenomenon implies the possibility of finding locations within the heated domain which have temperatures higher than the imposed boundary temperature at the wall. This phenomenon may occur in domains exposed to sudden changes in their wall temperature, if the domain has non-zero initial temperature time gradient, $\frac{\partial T}{\partial t}(0, x) = \dot{T}_o$, and if

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the thermal behavior of such a domain is described by heat conduction models other than the classical parabolic (diffusion) heat conduction model. Examples of these models are the hyperbolic, the dual-phase-lag and the microscopic models.

In the literature, the overshooting phenomenon has been investigated under the effect of the macroscopic wave, macroscopic dual-phase-lag, and microscopic parabolic heat conduction models [1–5]. The aim of the present work is to investigate the overshooting phenomenon in the microscopic hyperbolic heat conduction model. The parameters within which this phenomenon may appear are investigated. An initial-condition map within which this phenomenon is significant will be presented.

2. ANALYSIS

In a one-dimensional semi-infinite domain, the governing equations for the hyperbolic microscopic heat conduction model are given as

$$C_e \frac{\partial T_e}{\partial t} = -\frac{\partial q_e}{\partial x} - G(T_e - T_l) \tag{1}$$

$$C_l \frac{\partial T_l}{\partial t} = G(T_e - T_l) \tag{2}$$

$$\tau_F \frac{\partial q_e}{\partial t} + K \frac{\partial T_e}{\partial x} + q_e = 0 \tag{3}$$

Equations (1)–(3) are combined to yield the following equation in terms of T_l:

$$\frac{\partial^2 T}{\partial x^2} + \frac{C_l}{G} \frac{\partial^3 T}{\partial x^2 \partial t} = \tau_F \frac{C_e C_l}{KG} \frac{\partial^3 T}{\partial t^3} + \left[\frac{\tau_F (C_e + C_l)}{K} + \frac{C_e C_l}{KG} \right] \frac{\partial^2 T}{\partial t^2} + \left[\frac{(C_e + C_l)}{K} \right] \frac{\partial T}{\partial t} \tag{4}$$

where $T \equiv T_l$, where the subscript "l" is omitted for the sake of convenience. The thermal disturbance in the medium is induced by a suddenly imposed temperature at $x = 0$. As a result, Eq. (4) has the following boundary conditions:

$$T(t, 0) = T_w, \qquad T(t, x) \to T_o \qquad \text{as} \quad x \to \infty \tag{5}$$

In order for the overshooting behavior to appear, three initial conditions of non-zero finite values must be provided. These initial conditions are given as

$$T(0, x) = T_o, \qquad \frac{\partial T}{\partial t}(0, x) = \dot{T}_o, \qquad \frac{\partial^2 T}{\partial t^2}(0, x) = \ddot{T}_o \tag{6}$$

Now, using the dimensionless parameters defined in the Nomenclature, Eqs. (4)–(6) are rewritten as

$$\frac{\partial^2\theta}{\partial\delta^2}+D_1\frac{\partial^3\theta}{\partial\delta^2\partial\tau}=D_2\frac{\partial^3\theta}{\partial\tau^3}+(1+D_2)\frac{\partial^2\theta}{\partial\tau^2}+\frac{\partial\theta}{\partial\tau} \tag{7}$$

$$\theta(\tau,0)=1,\qquad \theta(\tau,\delta)\to 0 \quad \text{as} \quad \delta\to\infty \tag{8}$$

$$\theta(0,\delta)=0,\qquad \frac{\partial\theta}{\partial\tau}(0,\delta)=\dot{\theta}_o,\qquad \frac{\partial^2\theta}{\partial\tau^2}(0,\delta)=\ddot{\theta}_o \tag{9}$$

where

$$D_1=\frac{C_l}{G\tau_F},\qquad D_2=\frac{C_eC_l}{(C_e+C_l)\,\tau_F G}$$

Now, using the Laplace transformation technique, with the notation $L\{\theta(\tau,\delta)\}=W(s,\delta)$, Eqs. (7)–(9) assume the following solution:

$$W(s,\delta)=\left(\frac{1}{s}+\frac{\Omega}{\epsilon}\right)e^{-\sqrt{\epsilon}\,\delta}-\frac{\Omega}{\epsilon} \tag{10}$$

where

$$\Omega=-\frac{sD_2\dot{\theta}_o+D_2\ddot{\theta}_o+(1+D_2)\,\dot{\theta}_o}{1+D_1 s},$$

$$\epsilon=\frac{D_2s^3+(1+D_2)\,s^2+s}{1+D_1 s}$$

Equation (10) is inverted in terms of a Riemann–sum approximation as [5]

$$\theta(\tau,\delta)=\frac{e^{\gamma\tau}}{\tau}\left[\frac{1}{2}W(\gamma,\delta)+\mathrm{Re}\sum_{n=1}^{N}W\left(\gamma+\frac{in\pi}{\tau},\delta\right)(-1)^n\right] \tag{11}$$

and for faster convergence of Eq. (11), it has been shown that γ may be obtained from [5]

$$\gamma\tau=4.7 \tag{12}$$

3. RESULTS AND DISCUSSION

By tracing the spatial temperature distribution, as given in the inversion of Eq. (10), at early times, it is found that there are locations in the vicinity of the boundary which have temperatures larger than the imposed boundary temperature. This is what called the overshooting phenomenon.

Figures 1 and 2 represent two maps specifying the operating conditions within which the overshooting phenomenon may be observed. If the problem under consideration assumes finite values for $\dot{\theta}_o$ and $\ddot{\theta}_o$ such that the location of this state is below the specified curve for the given values of D_1 and D_2, then there is no possibility for the overshooting phenomenon to appear at any location or time. On the other hand, if the location of this state is above the curve, then this means that overshooting appears at certain locations and times.

Figure 1 shows the effect of D_2 on the overshooting map. It is clear that overshooting occurs at higher values of $\ddot{\theta}_o$, as compared to the values of $\dot{\theta}_o$, especially at small values of D_2. Also, it is clear that as D_2 increases, overshooting occurs at lower values of $\dot{\theta}_o$ and $\ddot{\theta}_o$. From its definition, D_2 increases as τ_F decreases. In the limit as $\tau_F \to 0$, then $D_2 \to \infty$. However, as $\tau_F \to 0$, the microscopic hyperbolic conduction model is reduced to the

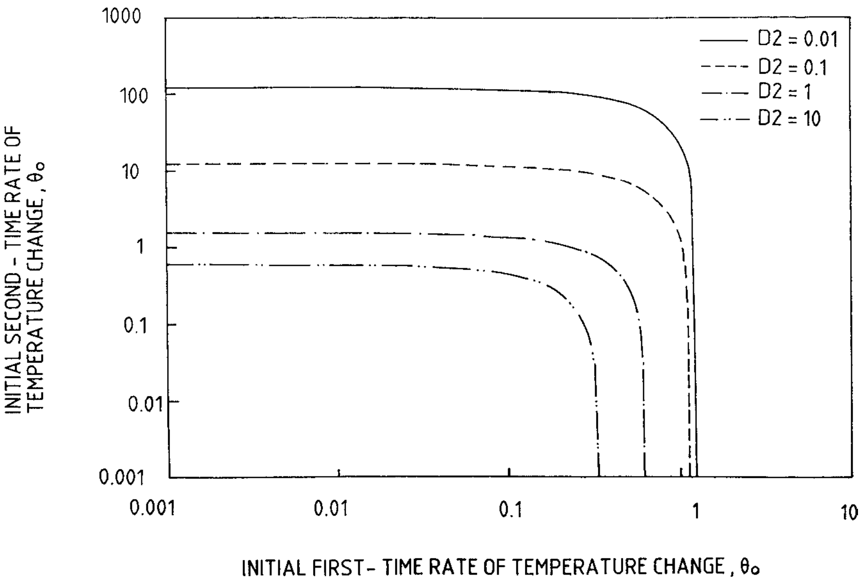

Fig. 1. A map for the overshooting phenomenon for different values of D_2. $D_1 = 1000$, $\tau = 1$.

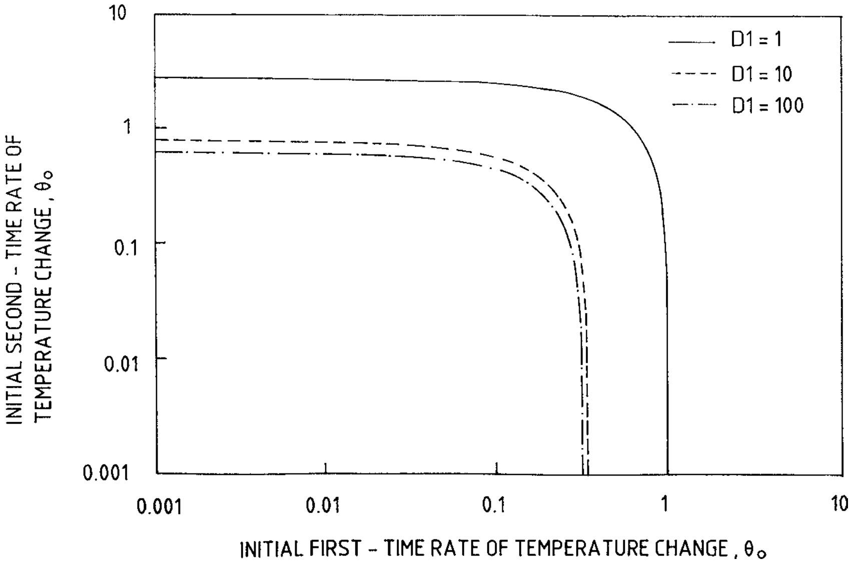

Fig. 2. A map for the overshooting phenomenon for different values of D_1. $D_2 = 10$, $\tau = 1$.

microscopic parabolic model. This implies that the overshooting phenomenon has a higher probability to appear in the parabolic microscopic heat conduction model than in the hyperbolic microscopic model. The effect of D_2 on the overshooting phenomenon is insignificant at large values of D_2.

Figure 2 shows the effect of D_1 on the overshooting phenomenon. As D_1 increases, overshooting appears at lower values of $\dot{\theta}_o$ and $\ddot{\theta}_o$. Also, overshooting may appear at values of $\dot{\theta}_o$ and $\ddot{\theta}_o$ having the same order of magnitude. As D_1 increases, the overshooting phenomenon is insensitive to the variation in D_1. From the results of Figs. 1 and 2, we may conclude that overshooting appears when $\frac{\dot{T}_o}{T_w - T_o}$ is of order of $10^{13} s^{-1}$ and when $\frac{\ddot{T}_o}{T_w - T_o}$ is of order of $10^{26} s^{-2}$. These two values represent extremely high initial time-derivatives of temperature which are very difficult to be produced in practical applications. This is the main obstacle to obtaining the temperature overshooting in the laboratory.

4. CONCLUDING REMARKS

The overshooting phenomenon in the microscopic hyperbolic heat conduction model is investigated. A map is presented of the region within which the overshooting phenomenon is significant. Four dimensionless parameters are found to control the overshooting phenomenon. These parameters are $\dot{\theta}_o$, $\ddot{\theta}_o$, $D_1 (= \frac{C_l}{G \tau_F})$, and $D_2 (= \frac{C_e C_l}{(C_e + C_l)\, \tau_F G})$. The overshooting phenomenon is enhanced as these four parameters increase. However, in order for overshooting to appear, higher values of $\ddot{\theta}_o$ are required than the values of $\dot{\theta}_o$. Also, overshooting has higher probability to appear in the parabolic microscopic model than in the hyperbolic microscopic model. Overshooting is difficult to observe in practical applications. This is due to two requirements which are the extremely high initial first and second time-derivatives of temperature and the very small spatial physical domain which is very close to the atomic level.

NOMENCLATURE

C	heat capacity, $\mathrm{J \cdot m^{-3} \cdot K^{-1}}$
G	electron-phonon coupling factor, $\mathrm{W \cdot m^{-3} \cdot K^{-1}}$
i	imaginary unit, $\sqrt{-1}$
K	thermal conductivity, $\mathrm{W \cdot m^{-1} \cdot K^{-1}}$
q	conduction heat flux, $\mathrm{W \cdot m^{-2}}$
s	Laplacian variable
t	time, s
T	temperature, K
$\dot{T}_o$	initial first time derivative of temperature, $\mathrm{K \cdot s^{-1}}$
$\ddot{T}_o$	initial second time derivative of temperature, $\mathrm{K \cdot s^{-2}}$
W	Laplace transform of T
x	spatial coordinate, m

Greek Symbols

α	thermal diffusivity, $K/(C_e + C_l)$
δ	dimensionless space coordinate, $x/\sqrt{\alpha\tau_F}$
θ	dimensionless temperature, $(T - T_o)/(T_w - T_o)$
$\dot{\theta}_o$	dimensionless initial first derivative of temperature, $(\dot{T}_o - T_o)/(T_w - T_o)$
$\ddot{\theta}_o$	dimensionless initial second derivative of temperature, $(\ddot{T}_o - T_o)/(T_w - T_o)$
τ	dimensionless time, t/τ_F
τ_F	relaxation time evaluated at Fermi surface, s
τ_q	relaxation time in heat flux, s
τ_T	relaxation time in temperature, s

Subscripts

e	electron
l	lattice
o	initial
w	wall

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